Cooling of a Compressed Bunch in the RFOFO Ring.

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Abstract

Front-end cooling simulation of a 'compressed' bunch is continued. Injected bunch for this stage is taken from a system considered in Ref. [1], which includes a proton driver, target station, phase rotation channel, and a ring bunch compressor. Parameters of this bunch are: transverse emittance 6.3 mm, longitudinal one 25 mm, yield 0.11 muons per incident proton. The further cooling is produced by means of RFOFO ring cooler of length 33 m with 200 MHz, 16 MeV/m accelerating system and liquid hydrogen absorbers. 14 m matching section between the bunch compressor and the cooler is considered also. After the cooling, parameters of the bunch are: transverse emittance 2.5 mm, longitudinal one 3.2 mm, yield 0.054 muons per proton.

1 Introduction

The high physics potential of a $\mu^+\mu^-$collider can only be realized if all its components achieve the required high performance [2]. In particular, it is desirable to collect all cooled muons in a single bunch, because for a given number of particles, luminosity of the collider is inversely proportional to the number of bunches. For this purpose, a system was considered in Ref. [1] including a proton driver, a linear accelerator for a phase rotation, and a bunch compressor based on a ring cooler with a wedge absorber destined for strong longitudinal cooling of the bunch. 36 MHz RF system with accelerating gradient about 6.4 MeV/m were used in that design both in the linear and the ring accelerators. It was shown that the system can produce a bunch containing about 0.11 muons per incident proton at transverse and longitudinal emittance about 6.3 mm and 25 mm correspondingly. The transverse emittance is still very large for a muon collider, therefore the further cooling is required. A usage of the RFOFO ring cooler for this is considered in this paper. Enough small initial longitudinal emittance of the bunch allows to apply 200 MHz RF system providing a high accelerating gradient. Note in this connection, that the large particle loss on previous stage was caused mostly by muon decay during a long time cooling required because of relatively small accelerating gradient in the bunch compressor. 200 MHz RFOFO ring cooler proposed in Ref. [3] and developed in Ref. [4] and [5] is used as a basis. Matching section between the compressor and the cooler is considered also. However, we do not include yet injection and extraction systems what is very special and challenging problem.

2 Matching Section

The output of Ref. [1] is used as the input for this simulation. Several projection of the incoming beam are presented in Figs. 1 and 2. It is seen from Fig.1 that the beam has some axial asymmetry both in space of transverse coordinates and momenta, however, there are no another significant correlations in the transverse part of the phase space. Some projections including axis of energy are shown in Fig.2. There is a correlation of energy with transverse coordinate and momentum that considerably increases energy spread. The origin of this effect is a dependence of revolution frequency of the compressor on transverse momentum at given total energy. Note that this correlation is helpful for a further cooling because it provides a better matching of the bunch with the cooler. Energy-time correlation (Fig.2, right) is small and can be suppressed after short drift in the matching channel.

In spite of mentioned correlations, the particle distributions differs from Gaussian not very strongly. R.m.s. characteristics of this distribution are given in column I of Table 1 (columns II and III will be commented later). Here and later, normalized emittance is treated as a simple production corresponding r.m.s. sizes of the beam divided on muon mass.

According the table, ratio of r.m.s. transverse momentum to the beam size $\sigma_{p_{x,y}}/\sigma_{x,y} \simeq 2.6$ MeV/c/cm what just corresponds to axial field of the compressor that is 1.75 T, and beta-function of about 74 cm. Beta-function of the RFOFO cooler in chosen injection point is 2 times less, i.e. an

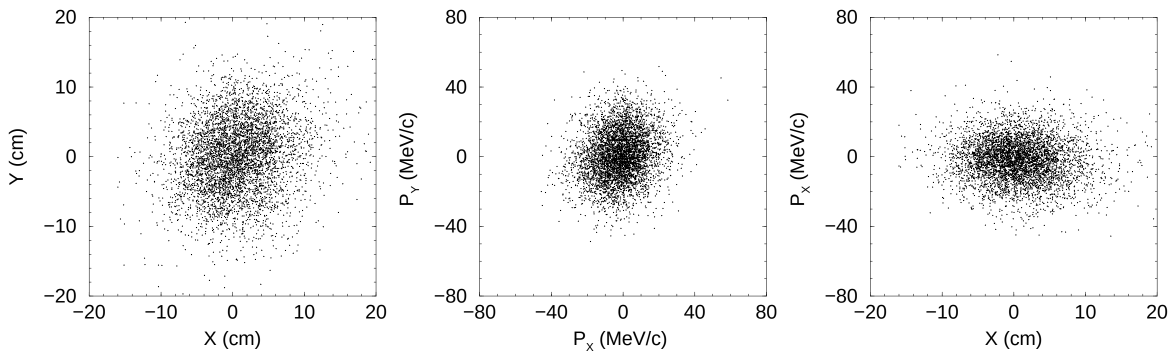

Figure 1: Projections of incoming beam in transverse phase space. Left – space of coordinates, center – transverse momenta, right – coordinate-momentum.

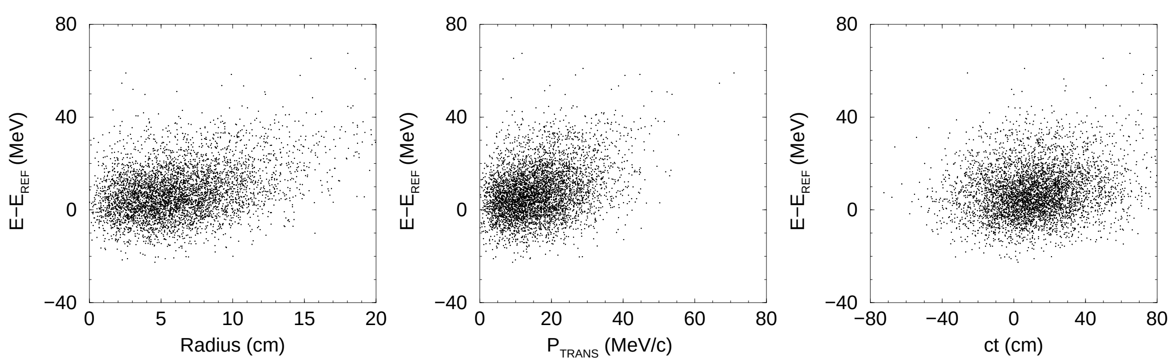

Figure 2: Correlation of energy and: particle radius (left); transverse momentum (center); time (right).

Table 1: R.m.s. beam parameters (I – injection, II – after the matching, III – after the cooling).

Parameter	I	II	III
Yield (μ/p)	.107	.101	.054
Horizontal size (cm)	4.83	3.63	2.32
Horizontal momentum (MeV/c)	12.2	18.4	12.1
Horizontal emittance (cm)	.558	.633	.265
Vertical size (cm)	5.28	3.53	2.21
Vertical momentum (MeV/c)	14.2	18.4	11.7
Vertical emittance (cm)	.709	.613	.244
Bunch length (ct, cm)	23.4	14.6	4.52
Energy spread (MeV)	11.3	18.1	7.57
Longitudinal emittance (cm)	2.51	2.50	.324

effective magnetic field is 2 times more. Therefore axial magnetic field should adiabatically increase in the matching section from 1.75 T to 3.5 T. A similar situation there is in longitudinal direction, where ratio σ_E/σ_{ct} is 2.7 times less then it is required for a matching with ~ 200 MHz accelerating system of the cooler. Therefore, a phase-rotating linac is required in the matching section being its main part.

Actually accelerating frequency of the RFOFO 203.4 MHz is taken in this design because it just corresponds to the cooler parameters at reference energy 220 MeV. It is rather close to a 'standard' frequency 201.25 MHz (see Ref.[6]), and can be easy corrected in future. The same frequency is used also in the matching section of length 14 m where 7 cavities are placed with a step of 2 m, each providing 4.8 MV of voltage. Axial magnetic field using for transverse focusing increases linearly from 1.75 T to 3.5 T. The field is introduced analytically, and coils are not designed yet because it is mostly engineering problem, strongly depending on injection system (and point) of the ring which is not considered in this note.

The beam parameters after the matching are given in Table 1, column II, and its phase space is plotted in Figs. 3 and 4 (the same projections and scales as in Figs. 1 and 2). All the particles including longitudinal tails are shown on the plots; however, only the particles at time interval -25 cm $< ct <$ 50 cm are accounted at a calculation of the beam emittance, transmission, etc. (it is approximately a region which can be captured by the RFOFO ring cooler). At this way, the particle loss in the channel is about 6% whereas it is 1% if only decay loss is taken into account. Probably, this difference – 5% – is a maximal gain which could be achieved by an addition of higher RF harmonics in the matching section. Decrease of the frequency does not improve the situation also in spite of less nonlinearity of the acceleration field. The point is that there is another strong source of the nonlinearity – mentioned above dependence of longitudinal velocity on transverse coordinate and momentum at given energy (see Figs. 2 and 4). At the matching, it leads to a lengthening of the bunch, and the effect becomes more when the channel is longer what is typical at the decrease of frequency (and accelerating gradient). In particular, a using of 36 MHz linac for the phase rotation (frequency of the compressor) is certainly worse. It is seen also that the beam obtains an axial symmetry after the matching but energy-radius and energy-transverse momentum correlations became stronger because of more magnetic field.

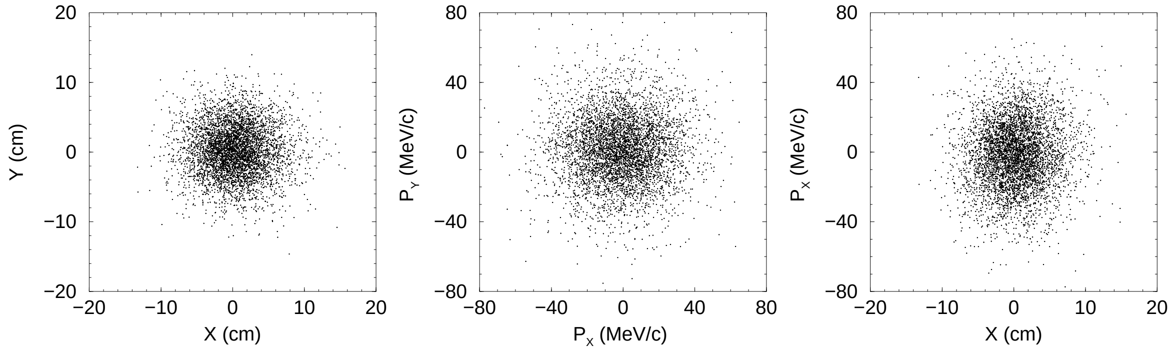

Figure 3: Projections of the beam after matching in transverse phase space. Left – usual space, center – space of transverse momenta, right – coordinate-momentum space.

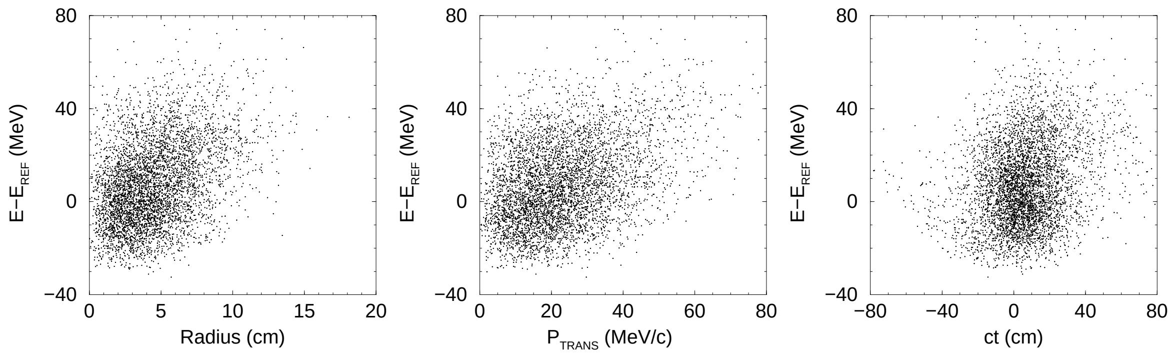

Figure 4: Correlation of energy and: particle radius (left); transverse momentum (center); time (right).

3 RFOFO Ring

RFOFO ring described in Ref. [4] is used for a further cooling of the muon bunch. The ring consists of 12 cells shown in Fig.5. The cell includes 2 solenoid coils with opposite currents, 5 cavities, and liquid hydrogen wedge absorber. The coils are tilted on ± 52 mrad to create vertical component

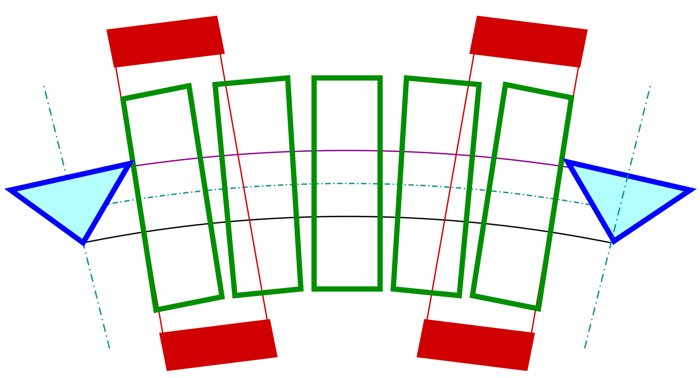

Figure 5: Schematic of RFOFO cooling cell).

Table 2: Parameters of the Cooler

Inner radius of coils	77 cm
Outer radius of coils	88 cm
Coil length	50 cm
Current density	± 95.27 A/mm^2
Tilting angle of the coil	± 52 mrad
Accelerating frequency	203.4 MHz
RF harmonic number	25
Accelerating gradient	16 MeV/m
Synchronous phase	33°
Absorber thickness at the center	48.2 cm
Energy loss at the center	15.1 MeV
Gradient ox energy loss dE/dy	1.21 MeV/cm

of the field. Period length is 275 cm along the centerline of the coils that is a circle of radius $R_0 = 275 \times 12/(2\pi) \simeq 525.21$ cm. Therefore circumference of the ring is 33 m along this line. However, the reference particle of energy 220 MeV has shorter orbit of length 32.32 m as it was shown in Ref. [4]. Space between the centers of the absorber and the nearest solenoid coil is 55 cm. Another parameters of the ring are listed in Table 2.

Evolution of the beam parameters at the cooling is plotted in Fig.5. Besides, some beam characteristics in the beginning of the cooling and after 10 turns are listed in Table 1, columns II and III. Muon yield is defined as a number of muons in the bunch per incident proton:

$$Y = \mu/p.$$

Merit factor is simply ratio of the yield to 6D phase volume:

$$M.F. = Y/\varepsilon_6.$$

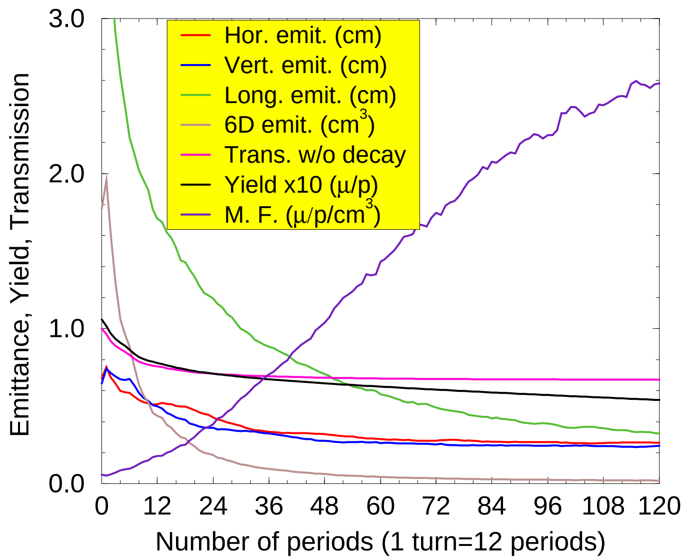

Figure 6: Evolution of the beam parameters at the cooling.

In contrast with traditional definition, we do not normalize this factor on its initial value because initial longitudinal distribution (after the matching) is not Gaussian, that is its emittance is rather arbitrary depending on used window. It explains also some discrepancy of initial longitudinal emittance in the table and plot, because the cut -25 cm $< ct <$ 50 cm is not used at the plotting. Therefore there is considerable particle loss approximately during 1st turn. However, after that the particles are lost almost only because of decay. With this loss taken into account, transmission of the ring is about 51% and average transverse cooling factor is 2.45 after 10 turns. Longitudinal cooling is stronger so that corresponding cooling factor achieves 7.72 if truncated initial emittance from Table 2 is used. Phase space of the beam after the cooling is plotted in Figs. 7 and 8 (the same projections and scales as in Figs. 1 and 2). It is seen that parasitic correlations are not so strong, and the distribution is rather close to Gaussian.

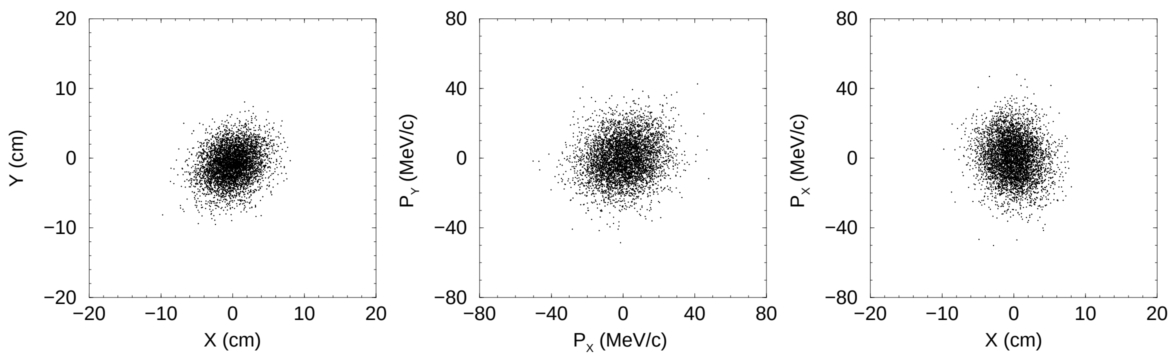

Figure 7: Projections of the cooled beam in transverse phase space. Left – usual space, center – space of transverse momenta, right – coordinate-momentum space.

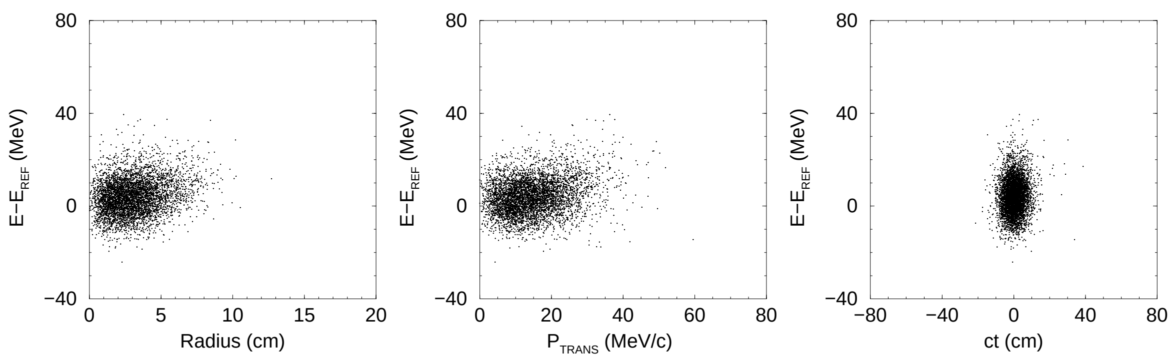

Figure 8: Correlation of energy and: particle radius (left); transverse momentum (center); time (right).

4 Conclusion

Front-end simulation with realistic field from target station to RFOFO ring cooler is performed. It is shown that the yield 0.054 muon on incident proton is achievable. The transmission of RFOFO cooler itself is 51%, and achievable emittances are about 2.5 mm in transverse direction and 3.2 mm in longitudinal one. These parameters are acceptable for a next steps of the cooling, e.g. by means of Li lenses.

References

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